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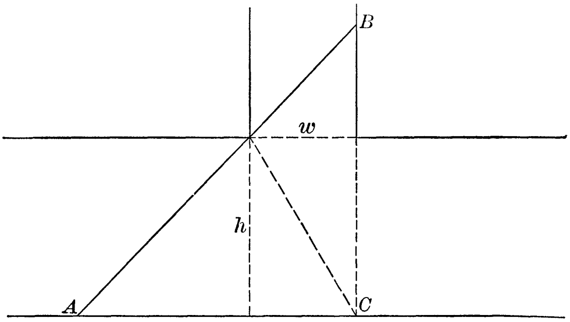
SOLUTIONS OF PROBLEMS.

ALGEBRA.

392. Proposed by H. PRIME, Boston, Mass.

The floor and ceiling of a room h feet high are parallel and horizontal. In the middle of the ceiling is a vertical circular tube w feet in diameter extending upward from the ceiling. From the room the longest possible inflexible rod is put up the tube. When the rod is in contact with the opposite sides of the tube and the floor, what is the expression for the tangent of the acute angle made by the rod and the floor in terms of h and w, considering the rod as an air line? To be solved without the calculus. (From The Maine Farmers' Almanac, 1913.)

I. Solution by H. E. Trefethen, Colby College.



If AB is the rod, $AC = x$ its projection on the floor, and $BC = y$, then

$$x^2 + y^2 = AB^2 = \text{minimum}, \qquad w/x + h/y = 1. \tag{1, 2}$$

Put $w/x = mu$, $x^2 = mu^{-2}$, $h/y = nv$, $y^2 = nv^{-2}$. Then $m = w^{\frac{2}{3}}$, $n = h^{\frac{2}{3}}$. Hence (1, 2) become $w^{\frac{2}{3}}u^{-2} + h^{\frac{2}{3}}v^{-2} = \text{minimum}$, $w^{\frac{2}{3}}u + h^{\frac{2}{3}}v = 1$.

Now

$$(w^{\frac{2}{3}}u^{-2} + h^{\frac{2}{3}}v^{-2})/(w^{\frac{2}{3}} + h^{\frac{2}{3}}) \geqq [(w^{\frac{2}{3}}u + h^{\frac{2}{3}}v)/(w^{\frac{2}{3}} + h^{\frac{2}{3}})]^{-2}. \tag{3}$$

For if

$$w^{\frac{2}{3}}/(w^{\frac{2}{3}} + h^{\frac{2}{3}}) = p, \qquad h^{\frac{2}{3}}/(w^{\frac{2}{3}} + h^{\frac{2}{3}}) = q, \quad \text{then} \quad p + q = 1, \tag{4}$$

and if

$$u/(pu + qv) = r, \qquad v/(pu + qv) = s, \quad \text{then} \quad pr + qs = 1, \tag{5}$$

and if we divide both sides of (3) by the second member, then (3) reduces to $pr^{-2} + qs^{-2} \geqq 1$, which is true since

$$1 - r^2 \geqq 2r^2(1 - r) \quad \text{and} \quad 1 - s^2 \geqq 2s^2(1 - s).$$

And hence

$$r^{-2} - 1 \geqq 2(1 - r) \quad \text{and} \quad pr^{-2} - p \geqq 2p - 2pr, \tag{6}$$

$$s^{-2} - 1 \geqq 2(1 - s) \quad \text{and} \quad qs^{-2} - q \geqq 2q - 2qs. \tag{7}$$

Adding (6, 7) and reducing by (4, 5) we have $pr^{-2} + qs^{-2} \geqq 1$.

Thus (3) is established. Therefore, since the denominators and the second numerator are constants, $w^{\frac{2}{3}}u^{-2} + h^{\frac{2}{3}}v^{-2} =$ minimum when (3) is an equality, that is, when $u = v$. Hence $w/x = h^{\frac{1}{3}}/y$ and

$$y/x = h^{\frac{1}{3}}/w^{\frac{1}{3}} = \tan BAC \text{ as required.}$$

II. Solution by A. H. Holmes, Brunswick, Me.

A maximum length of pole would move with equal freedom on floor and side of tube, the condition being that the ratio between the part of the pole below the tube to the tangent of the angle it makes with the floor is equal to the ratio between the part of the pole within the tube and the tangent of the angle made by the extremity of the pole with the side of the tube, which is the cotangent of the angle made by the pole with the floor. Let h be the perpendicular height of the ceiling from the floor, w the diameter of the tube, and θ the acute angle made by the pole with the floor. Then we have

$$\frac{h}{\sin\theta} : \tan\theta = \frac{w}{\cos\theta} : \cot\theta.$$

Hence $\tan\theta = \sqrt[3]{h/w}$.

393. Proposed by H. E. Trefethen, Waterville, Maine.

Expand $\log[(\sin x)/x]$ in terms of x.

I. Solution by F. M. Morgan, Dartmouth College.

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots.$$

Hence

$$\frac{\sin x}{x} = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \cdots.$$

Now let

$$\frac{x^2}{3!} - \frac{x^4}{5!} + \frac{x^6}{7!} - \cdots = \gamma.$$

Then

$$\frac{\sin x}{x} = 1 - \gamma, \quad \text{and} \quad \log(1-\gamma) = -\gamma - \frac{\gamma^2}{2} - \frac{\gamma^3}{3} - \cdots.$$

Hence

$$\log[(\sin x)/x] = -\left(\frac{x^2}{3!} - \frac{x^4}{5!} + \cdots\right) - \left(\frac{x^2}{3!} - \frac{x^4}{5!} + \cdots\right)^2 - \cdots$$

$$= -\frac{x^2}{6} - \frac{x^4}{180} - \frac{x^6}{2835} - \cdots.$$

II. Solution by A. H. Holmes, Brunswick, Me.

$$\frac{d}{dx}\log[(\sin x)/x] = \cot x - \frac{1}{x}.$$